UESE(III) — Phy
(CC – 5)

2024(Backlog)

Time : 3 hours

Full Marks : 60

Pass Marks : 24

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A

(Compulsory)

1. Answer **all** questions of the following : 1×10 = 10

   (a) The value of $\Gamma\left(-\dfrac{1}{2}\right)$ is _____ .

   (b) Write the value of $\sum_{k=0}^{\infty} P_k(x)$.

   (c) $\dfrac{d}{dx}(x^n J_n) = $ _____ .

(Turn over)

(d) The series $\frac{1}{x} + 1 + x + x^2 + x^3 + \ldots$ is a power series or not ?

(e) If $f(x) = x^2, -\pi \le x \le \pi$ then in its series the co-efficient $b_n =$ _____.

(f) The Hermite polynomial $H_1(x) =$ _____.

(g) The Laguerre polynomial $L_n(0) =$ _____.

(h) The vale of $\beta(2, 3) =$ _____.

(i) For the Bessel's equation the regular singular point is at _____.

(j) The Legendre's polynomial $P_0(x) =$ _____.

2. What are Ordinary and Singular points ? Explain in brief.   5

OR

What are Dirichlet conditions ?

Group – B

Answer any **three** questions of the following :

$15 \times 3 = 45$

3. Find the series general solution of the differential equation :

$$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n + 1)y = 0$$

4. Find the series solution of the differential equation :

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$$

5. Represent the function $f(x) = x, -\pi < x < \pi$ in the form of Fourier series, and show that :

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots = \frac{\pi}{4}$$

6. Define Gamma and Beta functions and show that :

$$\beta(m, n) = \frac{\Gamma(m)\ \Gamma(n)}{\Gamma(m + n)}.$$

7. What is Laplace's equation ? Find its solution in spherical polar coordinate system.

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